Variational Method for Multiconductor Coupled Striplines with Stratified Anisotropic Media

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Abstract —**The applicability of the variational method is extended to the general structure of multiconductor coupled striplines with stratified uniaxially anisotropic substrates. Numerical examples are presented for the propagation constants as well as the characteristic impedances of various types of multicoupled striplines. Numerical computations are performed very accurately to provide sufficient precision even for tight coupling between strips of the multiconductor system; the basis functions used in the calculation are increased up to ten. Accurate numerical results reveal the mode coupling in the multimode propagation.**

I. Introduction

Various types of planar transmission lines have recently been investigated [1]–[22] from the point of view of applications not only in microwave and millimeter-wave transmission lines but also in interconnections between VLSI devices. Here highly accurate analysis is required, especially for microelectronic packaging with high densities, where crosstalk problems are serious. There have been a number of analytical methods for these transmission lines. Among them the variational method has been successfully applied, because it provides not only high-precision solutions but also the upper or lower bound, and the accuracy of the solution can be improved systematically by minimizing or maximizing the value. Stationary expressions have been reported for various types of transmission lines, including single and coupled striplines [1], [2], [7]–[9], [16], [31], broadside-coupled striplines [18], [19], [32], coplanar striplines [20], and coplanar waveguide [21], [22] with isotropic/anisotropic media. However a variational method for more than three coupled striplines has not yet been presented. Also, characteristic impedances of more than three coupled striplines have not been investigated in any publications.

In this paper, the applicability of the variational method is extended to the general structure of multiconductor coupled striplines with uniaxially anisotropic media (Fig. 1), which includes coupled microstrips, overlaid strips, and double-layered (suspended) strips. Numerical examples are presented for the propagation constants as well as the characteristic impedances of various types of multiconductor coupled striplines. The frequency-dependent solutions, which can be obtained by extending the procedure of the striplines with fewer than three conductors, are presented and are compared with the quasi-static values. Numerical computations have been carried out by applying the Ritz procedure to the variational expressions and by using the Galerkin procedure for the frequency-dependent solutions. The source quantities, the charge and current distributions on the strip, are represented in terms of the appropriate basis functions in these computations. Tight coupling between strips used for interconnections between VLSI devices with high densities markedly deforms the source distributions on the strip, especially in the region where the mode coupling occurs. Special care must be taken to obtain sufficient precision for the tight coupling case. Preliminary computations are performed by using a sufficient number of basis functions (up to ten terms) to investigate the accuracy of the numerical method. Also, the numerical data of special cases are compared with the available exact analytical solutions.

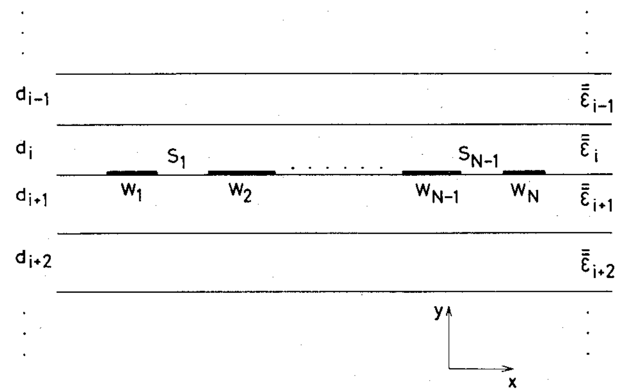

Fig. 1. General structure of N-coupled striplines with anisotropic substrates.

II. Theory

A. Quasi-Static Equations of Multiconductor Transmission Lines

The quasi-static characteristics of N-conductor coupled striplines can be described in general by N propagation constants and N^2 characteristic impedances. This section shows how these propagation parameters can be expressed

Manuscript received November 23, 1987; revised September 8, 1988. This work was supported by the SANEYOSHI Scholarship Foundation.
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IEEE Log Number 8825397.

0018-9480/89/0300-0484$01.00

in terms of the line constants. The characteristic impedances of N-conductor coupled striplines are defined as an extension of those for striplines with fewer than three conductors.

The basic equations for N-conductor TEM transmission lines are the generalized telegrapher equations [23], [26]–[30], which can be expressed as

$$-\frac{dV}{dz}=\bar{\bar{Z}}I$$

$$-\frac{dI}{dz}=\bar{\bar{Y}}V \tag{1}$$

$$V^T=[V_1,V_2,\cdots,V_N]$$

$$I^T=[I_1,I_2,\cdots,I_N] \tag{2}$$

where T denotes a transpose and V_i and I_i are the voltages and currents on the ith conductor, respectively. For lossless lines with uniaxially anisotropic media, one has

$$[\bar{\bar{Z}}]_{ij}=[\bar{\bar{Z}}]_{ji}=j\omega L_{ij}$$

$$[\bar{\bar{Y}}]_{ij}=[\bar{\bar{Y}}]_{ji}=j\omega C_{ij} \tag{3}$$

where L_{ii} and C_{ii} are the self-inductances and self-capacitances per unit length, and L_{ij} and C_{ij} $(i\neq j)$ are the mutual inductances and capacitances per unit length, respectively.

Assuming a z variation of the form $V_i(z)=V_{i0}e^{-j\beta z}$ for the voltages and $I_i(z)=I_{i0}e^{-j\beta z}$ for the current, the differential equations (1) reduce to the following eigenvalue equation:

$$\left[\bar{\bar{Z}}\bar{\bar{Y}}+\beta^2\bar{\bar{U}}\right]V=0 \tag{4}$$

where $\bar{\bar{U}}$ is the unit matrix of order N. For (4) to yield nontrivial solutions, the determinant of the coefficient matrix must be zero. This gives the characteristic equation for β, and it gives $2N$ roots:

$$\beta=\pm\beta_1,\pm\beta_2,\cdots,\pm\beta_N. \tag{5}$$

The general solutions for the voltage on the ith line can be written as

$$V_i=\sum_{k=1}^{N}R_{ik}\left(A_ke^{j\beta_kz}+B_ke^{-j\beta_kz}\right)\qquad(i=1,2,\cdots,N) \tag{6}$$

where A_i and B_i are constants, and R_{ik} is the ratio of the voltage of the ith strip V_i to that of the first strip V_1 for the mode k:

$$R_{ik}=\frac{V_i}{V_1}\qquad\text{for }\beta=\beta_k\qquad(k=1,2,\cdots,N). \tag{7}$$

The currents I_i are obtained by substituting (6) into (1):

$$I_i=\sum_{k=1}^{N}\frac{1}{Z_{ik}}R_{ik}\left(-A_ke^{j\beta_kz}+B_ke^{-j\beta_kz}\right) \tag{8}$$

where Z_{ik} is the characteristic impedance of line i for mode k and is given by

$$Z_{ik}=\frac{R_{ik}}{\beta_k}\cdot\frac{|\bar{\bar{Z}}|}{\sum_{j=1}^{N}R_{jk}\tilde{Z}_{jk}} \tag{9}$$

where $\tilde{Z}_{ij}$ and $|\bar{\bar{Z}}|$ are, respectively, the cofactor and the determinant of the impedance matrix $\bar{\bar{Z}}$.

B. Variational Expressions for the Quasi-Static Parameters

It became evident through the preceding discussion that the quasi-static characteristics, i.e., the phase constants β_k and the characteristic impedances Z_{ik}, for lossless N-coupled lines can be described in terms of line inductances L_{ij} and capacitances C_{ij} (eqs. (5) and (9)). L_{ij} are obtained by using the self- and mutual capacitances C_{ij} of the case without substrates. The line capacitances C_{ij} should be calculated precisely to obtain highly accurate line characteristics. In this section a method based on the variational principle is explained.

The capacitances are defined as

$$Q=CV \tag{10}$$

where

$$C=\begin{bmatrix}C_{11}&C_{12}&\cdots&C_{1N}\\C_{12}&C_{22}&\cdots&C_{2N}\\\vdots&&&\vdots\\C_{1N}&C_{2N}&\cdots&C_{NN}\end{bmatrix} \tag{11}$$

and the transpose of Q is

$$Q^T=[Q_1,Q_2,\cdots,Q_N] \tag{12}$$

where Q_i is the total charge on the strip i. Equation (10) can be solved for V:

$$V=DQ \tag{13}$$

where D is the compliance matrix [9], [16], the inverse matrix of the capacitance matrix

$$D=C^{-1}$$

$$D=\begin{bmatrix}D_{11}&D_{12}&\cdots&D_{1N}\\D_{12}&D_{22}&\cdots&D_{2N}\\\vdots&&&\vdots\\D_{1N}&D_{2N}&\cdots&D_{NN}\end{bmatrix} \tag{14}$$

In the following, variational expressions for the compliance matrix elements are derived for the general structure of asymmetrical N-coupled striplines with uniaxially anisotropic media (Fig. 1), which enables one to calculate the quasi-static characteristics, β_k and Z_{ik}, accurately. The formulation can be performed by extending the procedure used for three-line coupled striplines [16], and the hypothetical sidewalls [31], [32] are not introduced in the formulation.

The tensor permittivity of the ith layer of the stratified anisotropic media is given as

$$\bar{\bar{\epsilon}}_i=\begin{bmatrix}\epsilon_{ixx}&\epsilon_{ixy}\\\epsilon_{ixy}&\epsilon_{iyy}\end{bmatrix}\epsilon_0. \tag{15}$$

The potential distribution at the strip plane can be obtained by using the extended version of the method in [9], [16], and [22]:

$$\phi(x)=\int_{-\infty}^{\infty}\int_{0}^{\infty}G(\alpha;x|x')\sigma(x')\,d\alpha\,dx' \tag{16}$$

where $\sigma(x)$ is the charge distribution on the strips and

$$G(\alpha;x|x')=\frac{1}{\pi\epsilon_0\alpha}\cdot\frac{1}{Y_U(\alpha)+Y_L(\alpha)}\cos\{\alpha(x-x')\}. \tag{17}$$

Y_U and Y_L can be obtained by utilizing the simple recurrent relation proposed in [22].

The function $\phi(x)$ should be constant over the strip conductors:

$$\phi(x)=V_i \qquad (i=1,2,\cdots,N). \tag{18}$$

We consider the following sets of excitations:

$$Q_i\neq 0 \qquad Q_j=0 \qquad (j\neq i) \tag{19a}$$

to determine D_{ii}, and

$$Q_i=Q_j\neq 0 \qquad Q_k=0 \qquad (k\neq i,j) \tag{19b}$$

to determine D_{ij}. From (16), (18), and (19a), we obtain

$$\begin{aligned}V_1Q_1&=V_1\int_{W_1}\sigma(x)\,dx\\&=\int_{W_1}\int_{-\infty}^{\infty}\int_{0}^{\infty}\sigma(x)G(\alpha;x|x')\sigma(x')\,d\alpha\,dx'\,dx\\0&=V_2\int_{W_2}\sigma(x)\,dx\\&=\int_{W_2}\int_{-\infty}^{\infty}\int_{0}^{\infty}\sigma(x)G(\alpha;x|x')\sigma(x')\,d\alpha\,dx'\,dx\\&\;\;\vdots\\0&=V_N\int_{W_N}\sigma(x)\,dx\\&=\int_{W_N}\int_{-\infty}^{\infty}\int_{0}^{\infty}\sigma(x)G(\alpha;x|x')\sigma(x')\,d\alpha\,dx'\,dx\end{aligned} \tag{20}$$

by utilizing

$$\begin{aligned}Q_1&=\int_{W_1}\sigma(x)\,dx\neq 0\\Q_i&=\int_{W_i}\sigma(x)\,dx=0 \qquad (i=2,\cdots,N).\end{aligned} \tag{21}$$

Therefore, we get

$$\begin{aligned}D_{11}&=\frac{V_1}{Q_1}\bigg|_{Q_i=0\ (i\neq 1)}\\&=\iint_{-\infty}^{\infty}\int_{0}^{\infty}\sigma(x)G(\alpha;x|x')\sigma(x')\,d\alpha\,dx'\,dx/Q_1^2.\end{aligned} \tag{22}$$

It can easily be shown that (22) has a stationary property and that it gives an upper bound to the exact value of D_{11}. Expressions for the diagonal elements $D_{22},\cdots,D_{NN}$ can be obtained similarly. By using an analogous procedure applied to (16), (18), and (19b), we obtain an expression for $D_{ii}+2D_{ij}+D_{jj}$, which resembles (22), and it provides the off-diagonal elements, e.g. D_{ij}.

C. Hybrid-Mode Analysis Based on Galerkin's Method

The hybrid-mode formulation can be performed for asymmetrical N-coupled striplines with uniaxially anisotropic substrates cut with their planar surface perpendicular to the optical axis by extending the procedure used in [8], [9], and [16]. The electromagnetic fields are expressed in terms of the current densities $i_x(x)$ and $i_z(x)$ on the strip conductors, and no approximations for simplification are included in the formulation procedure. The method of solution for the propagation constants is based on Galerkin's procedure with appropriate basis functions used to express the unknown currents $i_x(x)$ and $i_z(x)$.

There is some ambiguity in the definition of the frequency-dependent characteristic impedances of striplines because of the hybrid-mode propagation, and several possible definitions have been proposed for single and two coupled striplines, e.g., the power–current [8], [24], voltage–current [25], and power–voltage [24] definitions. However, for the case of more than three coupled lines, the characteristic impedances based on the power flow do not have much meaning, because the total power flow cannot be allocated to the individual strips properly. The definition chosen here is on the voltage–current basis, i.e.,

$$Z_{ik}=\frac{V_{ik}}{I_{ik}} \tag{23}$$

where V_{ik} and I_{ik} are the voltage at the center of strip i and the total current on strip i for mode k, respectively.

III. Numerical Examples

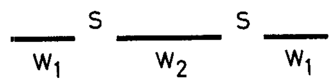
Numerical computations have been carried out by using the Ritz procedure for the quasi-static and the Galerkin procedure for the frequency-dependent hybrid-mode analysis. The accuracy of these computations depends on a reasonable choice of the basis functions which are used to represent the unknown quantites. These basis functions must incorporate the edge effect properly, they must be Fourier integrable analytically, and, especially for the cases of coupled multistrip lines, they must satisfy the excitation conditions (19) efficiently. We build up the following basis functions of the ith strip:

$$f_{ik}(x)=\frac{T_{k-1}\left\{\dfrac{2(x-C_i)}{W_i}\right\}}{\sqrt{1-\left\{\dfrac{2(x-C_i)}{W_i}\right\}^2}} \tag{24}$$

for the charge densities and the z components of the

TABLE I
NORMALIZED LINE CAPACITANCE OF THREE-LINE COUPLED STRIPS WITHOUT SUBSTRATE C_0/ϵ_0

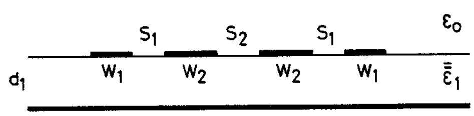
N	Even (Error %)	Odd (Error %)
1	2.1478 (14.61)	2.6605 (25.74)
2	2.4092 (4.22)	3.5433 (1.09)
3	2.5078 (0.30)	3.5592 (0.65)
4	2.5115 (0.15)	3.5808 (0.05)
5	2.5150 (0.01)	3.5812 (0.04)
6	2.5151 (0.01)	3.5822 (0.01)
7	2.5153 (0.00)	3.5822 (0.01)
8	2.5153 (0.00)	3.5823 (0.01)
9	2.5153 (0.00)	3.5824 (0.00)
10	2.5153 (0.00)	3.5824 (0.00)
Exact Value by Conformal Mapping (Appendix I)	2.5153	3.5825



$W_2/W_1 = 1.5$; $S/W_1 = 0.1$.

TABLE II
QUASI-STATIC CHARACTERISTICS OF FOUR COUPLED MICROSTRIPS ϵ_{eff}

N	mode 1	mode 3	mode 2	mode 4
1	9.0455	6.2829	7.1433	5.9049
2	9.0055	6.1216	6.8748	5.8572
3	9.0647	6.1126	6.8754	5.8388
4	9.0651	6.1135	6.8755	5.8391
5	9.0654	6.1135	6.8754	5.8392
6	9.0654	6.1135	6.8754	5.8392
7	9.0654	6.1135	6.8754	5.8392
8	9.0654	6.1135	6.8754	5.8392
9	9.0654	6.1135	6.8754	5.8392
10	9.0654	6.1135	6.8754	5.8392



$\epsilon_{1xx} = 9.4$, $\epsilon_{1yy} = 11.6$, $\epsilon_{1xy} = 0$, $W_2/W_1 = 1.2$, $S_1/W_1 = 0.2$, $S_2/W_1 = 0.1$, $d_1/W_1 = 1$.

currents, and

$$g_{ik}(x) = U_{k-1}\left\{\frac{2(x - C_i)}{W_i}\right\} \tag{25}$$

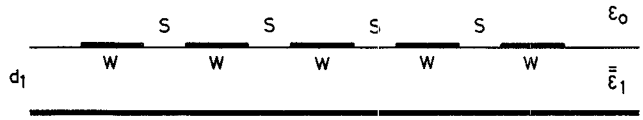
for the x components of the currents, where $T_k(x)$ and $U_k(x)$ are Chebyshev polynomials of the first and second kind, respectively, and C_i is the center of the ith strip. It should be noted that the integral of $f_{ik}(x)$ over the ith strip conductor becomes zero, except that of $f_{i1}(x)$, and the excitation requirements (19) are easily met by using (24). For the cases of multiple coupled striplines, many semi-infinite integrals should be calculated numerically. The convergence of the semi-infinite integrals becomes slower with the complete set of basis functions such as (24) and (25) than with the simpler set of functions [31]. However, it is possible to improve the rate of convergence of the integrals by the following procedure:

$$\int_0^\infty G\,d\alpha = \int_0^\infty (G - G_\infty)\,d\alpha + \int_0^\infty G_\infty\,d\alpha \tag{26}$$

where G_∞ is an approximation of G for large α. The

TABLE III
QUASI-STATIC CHARACTERISTICS OF FIVE COUPLED MICROSTRIPS ϵ_{eff}

N	mode 1	mode 3	mode 5	mode 2	mode 4
1	8.1066	5.9761	5.5511	6.6485	5.6719
2	8.0808	5.8585	5.5377	6.5208	5.6166
3	8.1093	5.8560	5.5345	6.5272	5.6123
4	8.1094	5.8563	5.5346	6.5273	5.6126
5	8.1095	5.8563	5.5346	6.5273	5.6126
6	8.1095	5.8563	5.5346	6.5273	5.6126
7	8.1095	5.8563	5.5346	6.5273	5.6126
8	8.1095	5.8563	5.5346	6.5273	5.6126
9	8.1095	5.8563	5.5346	6.5273	5.6126
10	8.1095	5.8563	5.5346	6.5273	5.6126



$\epsilon_{1xx} = 10$, $\epsilon_{1yy} = 10$, $\epsilon_{1xy} = 0$, $S/W = 0.2$, $d_1/W = 1$

TABLE IV
HYBRID-MODE SOLUTIONS OF FIVE-COUPLED MICROSTRIPS ϵ_{eff}

N	mode 1	mode 3	mode 5	mode 2	mode 4
1	8.3208	6.7330	5.7207	7.2167	6.1949
2	8.2151	5.8666	5.6126	6.5450	5.6533
3	8.1722	5.8649	5.5364	6.5431	5.6176
4	8.1719	5.8600	5.5359	6.5399	5.6142
5	8.1718	5.8600	5.5352	6.5399	5.6139
6	8.1718	5.8600	5.5352	6.5399	5.6139
7	8.1718	5.8600	5.5352	6.5399	5.6139
8	8.1718	5.8600	5.5352	6.5399	5.6139
9	8.1718	5.8600	5.5352	6.5399	5.6139
10	8.1718	5.8600	5.5352	6.5399	5.6139
Ref.[17]	8.305	5.868	5.560	6.533	5.630

$W = 1$(mm), $f = 1$(GHz).
Other dimensions are the same as in Table III.

second integral on the right can be expressed in closed form, while the first on the right converges rapidly compared to the integral on the left, and accurate values can be obtained numerically.

For the cases of multiple coupled striplines used for interconnections between VLSI devices with high densities, tight coupling between strips deforms the charge and current distributions on the strip to an unusual extent, which requires more basis functions to express unknown quantities. Some preliminary computations were performed to investigate the convergence of the calculated results with respect to the number of basis functions. The convergence of the quasi-static values is demonstrated in Table I, which shows the normalized line capacitances of three-line coupled strips without substrates or ground conductors, for which the exact analytical solutions can be obtained by conformal mapping (see the Appendix). It should be noted that (24) gives an upper bound to the compliances; hence the values in Table I are lower than the exact values. However as the number of basis functions increases, the values become larger approaching the exact value, and using more than five terms can afford rigorous solutions even for the very tight coupling cases considered here. Tables II and III demonstrate the convergence of the effective dielectric constants of the four-strip case with anisotropic substrate and of the five-strip case with

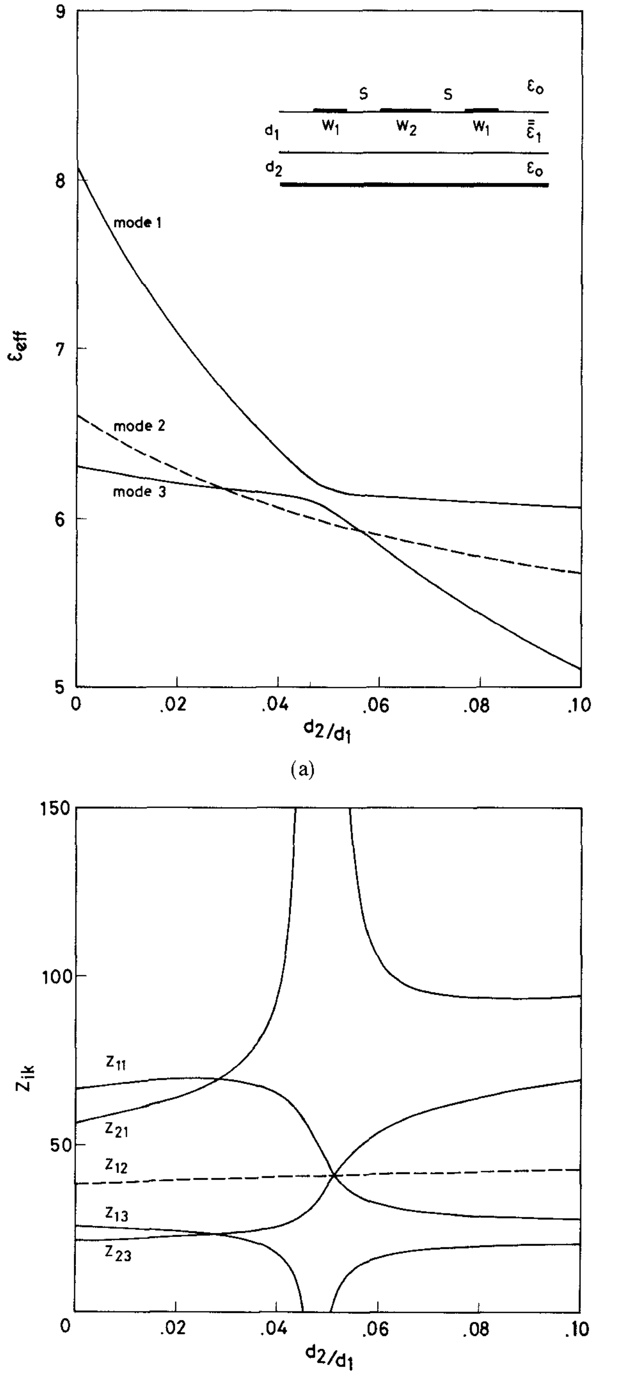


Fig. 2. Quasi-static characteristics of three coupled suspended strips. (a) Effective dielectric constants. (b) Characteristic impedances. $\epsilon_{1xx}=13$, $\epsilon_{1yy}=10.2$, $\epsilon_{2xx}=\epsilon_{2yy}=1$, $\epsilon_{ixy}=0$ $(i=1,2)$; $W_2/W_1=1.5$, $S/W_1=0.2$, $d_1/W_1=1$.

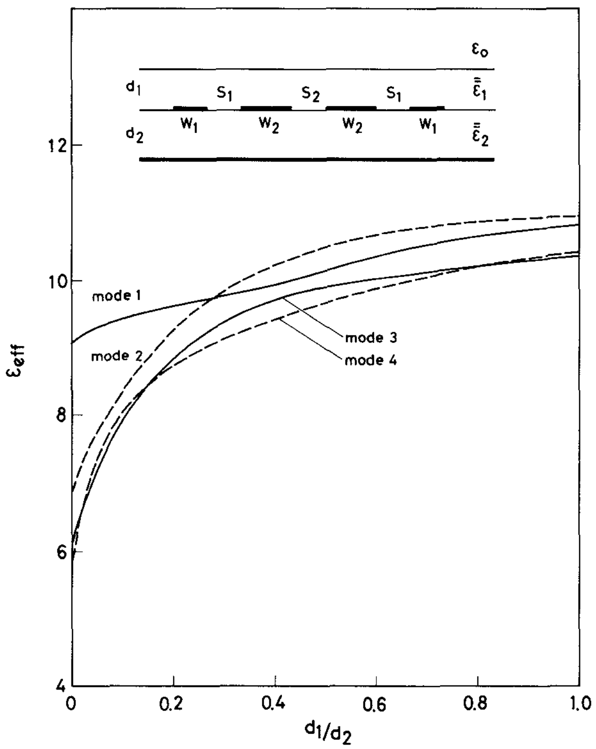


Fig. 3. Quasi-static characteristics of four coupled overlaid strips. $\epsilon_{1xx}=13$, $\epsilon_{1yy}=10.2$, $\epsilon_{2xx}=9.4$, $\epsilon_{2yy}=11.6$, $\epsilon_{ixy}=0$ $(i=1,2)$; $W_2/W_1=1.2$, $S_1/W_1=0.2$, $S_2/W_1=0.1$, $d_2/W_1=1$.

isotropic substrate, respectively. The exact analytical solutions are not available in these cases, but rapid convergence is observed.

Table IV demonstrates the convergence of the hybrid-mode values for five coupled striplines with the same dimensions as in Table III. The number of basis functions N, which are used to express both unknown currents, $i_x(x)$ and $i_z(x)$, is increased up to $N=10$ in these computations. The convergence of the hybrid-mode values is very fast

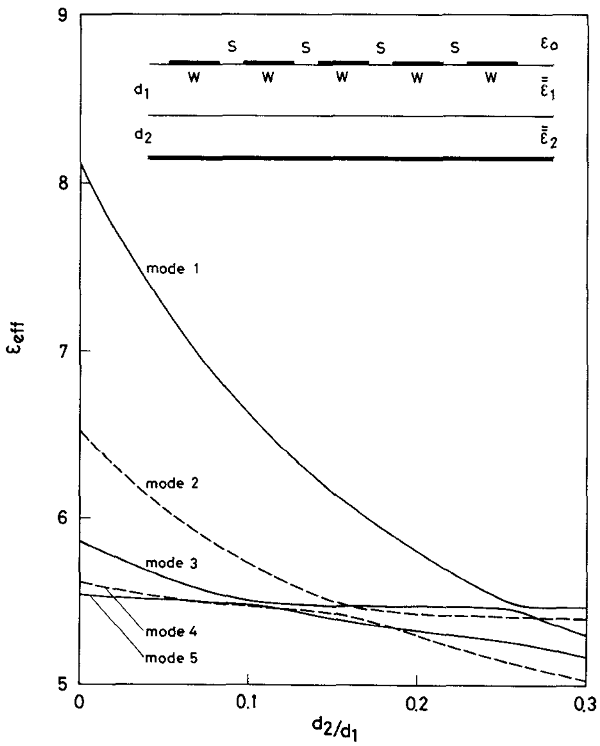
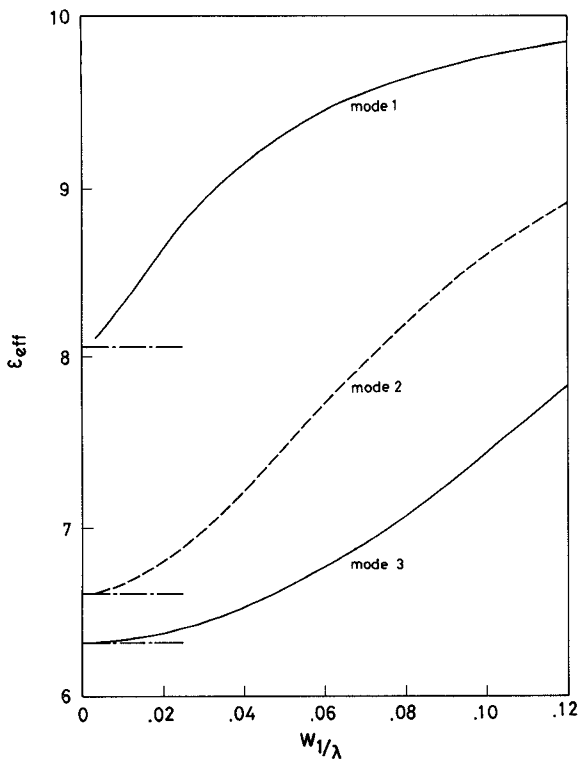
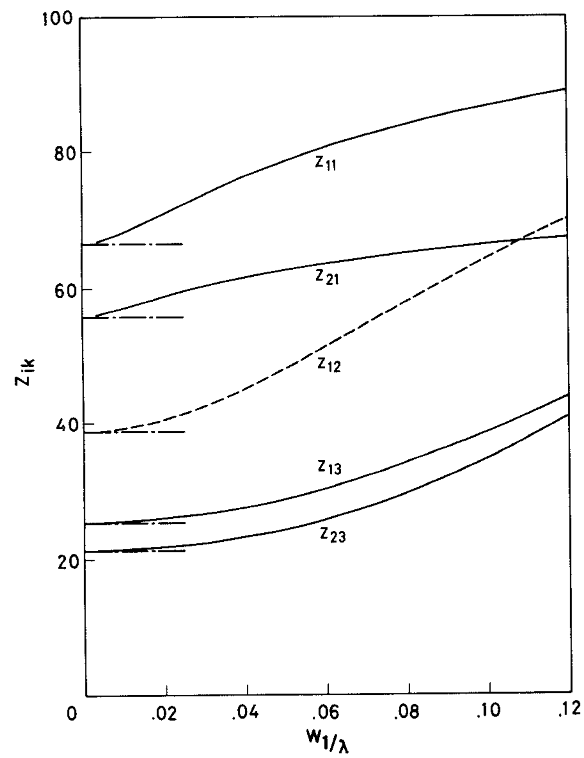


Fig 4. Quasi-static characteristics of five coupled double-layered strips. $\epsilon_{1xx}=10$, $\epsilon_{1yy}=10$, $\epsilon_{2xx}=2.6$, $\epsilon_{2yy}=2.6$, $\epsilon_{ixy}=0$ $(i=1,2)$; $S/W=0.2$, $d_1/W=1$.



(a)



(b)

Fig. 5. Dispersion characteristics of three coupled microstrips. (a) Effective dielectric constants. (b) Characteristic impedances. Dimensions are the same as in Fig. 2, except $d_2 = 0$. ———, ---- hybrid-mode; —·— quasi-static.

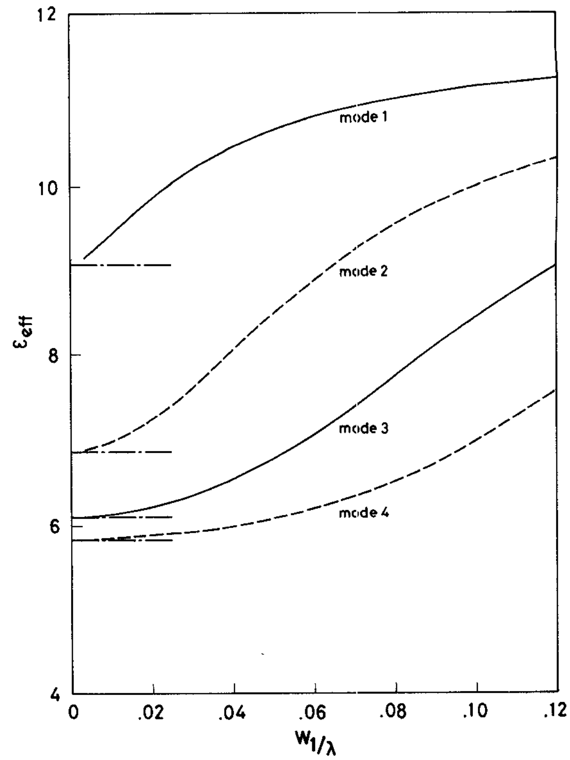


Fig. 6. Dispersion characteristics of four coupled microstrips. Dimensions are the same as in Fig. 3, except $d_1 = 0$. ———, ---- hybrid-mode; —·— quasi-static.

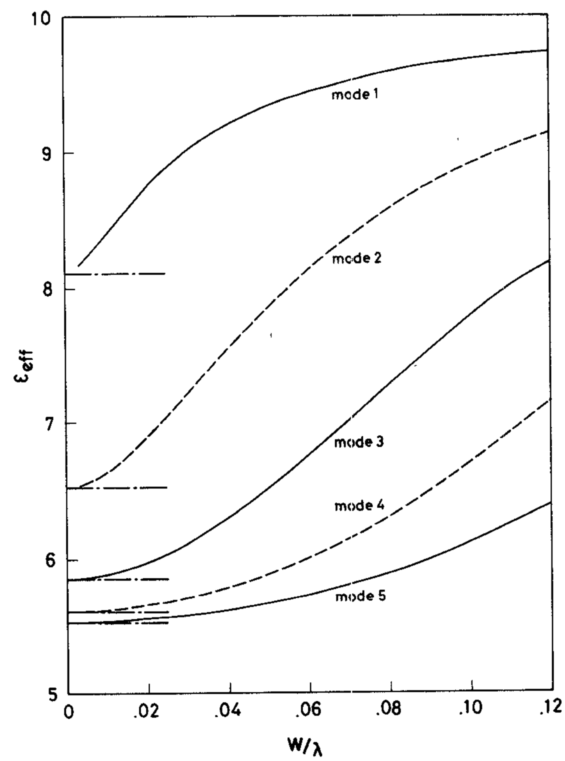


Fig. 7. Dispersion characteristics of five coupled microstrips. Dimensions are the same as in Fig. 4, except $d_2 = 0$. ———, ---- hybrid-mode; —·— quasi-static.

too. Also, the computed values are compared with those from [17] in Table IV.

Various types of striplines can be treated by the present method. Fig. 2 shows the line parameters of three coupled suspended strips with different spacing between the substrate and the ground conductor. Modes are designated for the microstrip case ($d_2/d_1=0$). Fig. 3 shows those of four coupled overlaid strips. Fig. 4 shows those of five coupled double-layered strips. Mode coupling is observed in these figures, e.g., the coupling between mode 1 and mode 3 of three coupled suspended strips (Fig. 2(a) and (b)) occurs in the region of spacing $d_2/d_1=0.05$, where the phase constants of the modes have close values but never coincide, which produces the unusual curves. Modes cannot be identified in these regions, and the characteristic impedances do not have definite values (Fig 2(b)).

The quasi-static and frequency-dependent characteristics are compared in Figs. 5–7. Fig. 5 shows the effective dielectric constants and the characteristic impedance of three coupled microstrips on an anisotropic substrate. It should be noted that the frequency-dependent values of characteristic impedances based on the above definition converge to the corresponding quasi-static values in lower frequency ranges precisely (Fig. 5(b)), showing the validity of the definition chosen. Also, it is observed that the frequency-dependent values of the effective dielectric constants converge precisely to the quasi-static values for the three, four, and five coupled strip cases in Fig. 5(a), Fig. 6, and Fig. 7, respectively.

IV. Conclusions

The variational method is presented for the propagation characteristics of the general structure of multiconductor coupled striplines with uniaxially anisotropic media. An accurate numerical procedure, which can provide sufficient precision even for tight coupling between strips used as interconnections between VLSI devices, is demonstrated. Numerical examples are presented for various types of multiple coupled striplines. Numerical results with high precision show for the first time the mode coupling in the multimode propagation, which causes the unusual curves. Also, the frequency-dependent values of the effective dielectric constants as well as the characteristic impedances, based on a properly chosen definition, converge to the corresponding quasi-static values in lower frequency ranges precisely, showing the accuracy of the method.

Appendix

The exact values of the line capacitances of symmetrical three-line coupled strips without substrates or ground conductors (Fig 8(a)) can be obtained by conformal mapping. Fig. 8(a)–(e) shows the series of transformations to obtain the analytical solution of the even mode. The determinantal equation for $k_2=p_2/q_2$ in Fig. 8(d) is

$$-F\left\{\arcsin\left(\frac{b_0}{c_0}\right),\, k_0\right\}+K(k_0)=F\{\arcsin(k_2),\, k_0\}$$

$$k_0=a_0/b_0$$

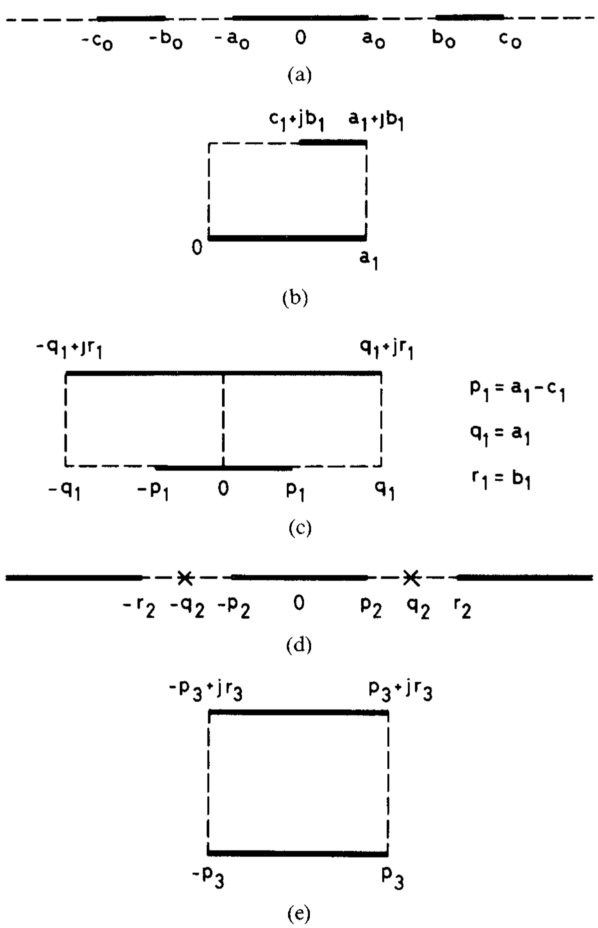


Fig. 8. A series of transformations for the even mode of symmetrical three coupled strips. ——— electric wall; ---- magnetic wall.

where $F(a,b)$ is the elliptic integral of the first kind and $K(k)$ is the complete elliptic integral of the first kind. Then, p_3 and r_3 in Fig. 8(e) can be determined as

$$p_3=CK(k_0k_2) \qquad r_3=CK'(k_0k_2)$$

$$C\colon \text{constant}.$$

The analytical solution of the odd mode can be obtained in similar fashion.

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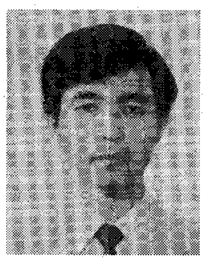

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